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**MEASUREMENT OF DIRECT MOMENT DERIVATIVES IN THE
PRESENCE OF STING PLUNGING**

**DÉTERMINATION EXPÉRIMENTALE DES DÉRIVÉES DE MOMENTS
AÉRODYNAMIQUES EN PRÉSENCE DU MOUVEMENT DE
PILLONNEMENT DU DARD SUPPORT**

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ABSTRACT

This paper presents a review and extension of methods for determining the effects of sting oscillations on the measurement of dynamic moment derivatives, based on linear as well as nonlinear models. It is shown that two of the linearized methods will reduce to the same result, applicable to low-lift configurations and suitable for on-line data reduction. The location of the effective axis of rotation is determined for a model executing planar oscillations in two degrees of freedom. The equations for the effect of plunging on the derivatives due to angular oscillation can be simplified by relating the sting deflection parameters to the co-ordinate of this axis. Therefore, both the correction of the measured derivatives and their transformation to the reference centre can be accomplished after a single measurement is made, namely the location of the effective axis. The requirements for performing sting plunging corrections for aircraft configurations at high angles of attack are discussed.

RÉSUMÉ

On présente une revue et extension des méthodes linéaires et non linéaires qui prennent en ligne de compte l'oscillation du dard support dans la détermination des dérivées de moments aérodynamiques instationnaires. On démontre que deux méthodes linéarisées se confondent et peuvent être employées dans le cas de portances faibles tout en se prêtant au dépouillement simultané. On détermine la position effective de l'axe de rotation d'une maquette mue d'une oscillation à deux degrés de liberté dans un plan. En reportant les paramètres de la translation du dard support à cette origine on simplifie les équations comportant l'effet du mouvement de pillonnement sur les dérivées du mouvement angulaire. Il s'en suit qu'après une seule détermination, c.a.d., celle de la position effective de l'axe, on est en mesure de purger les dérivées expérimentales et de les rattacher à l'origine. On traite de l'importance d'extraire l'ingérence du pillonnement lorsqu'un aéronef est à hautes incidences.

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SYMBOLS

Symbol	Definition
$\bar{c}$	reference length
C_L	lift coefficient
$C_{L\alpha}$	$\partial C_L / \partial \alpha$
C_{Lq}	$\partial C_L / \partial (q\bar{c}/V_\infty)$
C_m	pitching moment coefficient
C_{me}	$M_e / (q_\infty S\bar{c})$
C_{mq}	$\partial C_m / \partial (q\bar{c}/V_\infty)$
$C_{m\alpha}$	$\partial C_m / \partial \alpha$
$C_{m\dot{\alpha}}$	$\partial C_m / \partial (\dot{\alpha}\bar{c}/V_\infty)$
D, D_z	linear damping constants
I_y	pitch moment of inertia
K, K_z	linear spring constants
m	model mass or effective model and balance mass
M	aerodynamic pitching moment
M_e	harmonic excitation moment
M_p	moment applied to sting at the pivot
M_s	moment measured by sting bridge
q	body axis pitch rate
q_∞	dynamic pressure
S	reference area
t	time
V_∞	freestream velocity
x	axial body co-ordinate
x_{cg}	centre of mass location relative to pivot axis
z	wind-fixed pivot co-ordinate

SYMBOLS (Cont'd)

Symbol	Definition
Z	aerodynamic normal force
α	angle of attack
$\bar{\alpha}$	mean angle of attack
δ	increment
δ_{z0}	structural plunge damping $\delta_{z0} = \bar{c} D_z / (V_{\infty} m)$
$\delta_{\theta 0}$	structural pitch damping $\delta_{\theta 0} = \bar{c} D / (V_{\infty} I_y)$
Δ	amplitude
ζ	$z/\bar{c}$
θ	pitch perturbation angle
κ	$[I_y / (m \bar{c}^2)]^{1/2}$
μ	$m / (\rho_{\infty} S \bar{c})$
ξ	$x/\bar{c}$
ρ_{∞}	freestream density
τ	$\tau = (V_{\infty} / \bar{c}) / t$
Φ	phase angle between excitation moment and pitch oscillation
Φ_z	phase angle between translational and rotational motions
ω	angular velocity
$\bar{\omega}$	reduced frequency $\bar{\omega} = \omega \bar{c} / V_{\infty}$
$\bar{\omega}_{z0}$	natural frequency in pure translation $\bar{\omega}_{z0} = (\bar{c} / V_{\infty}) (K_z / m)^{1/2}$
$\bar{\omega}_{\theta 0}$	natural frequency in pure pitching $\bar{\omega}_{\theta 0} = (\bar{c} / V_{\infty}) (K / I_y)^{1/2}$
Ω	frequency dependence of structural damping

CONTENTS (Cont'd)

Symbol	Definition
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Subscripts

a	accelerometer
c	centre of rotation
cg	centre of mass
f	pivot or flexure
OFF	wind off
ON	wind on
z	translation
z	translation rate
o	single degree of freedom
θ	pitch

Superscripts

($\cdot$)	partial differentiation with respect to time
($\bar{\cdot}$)	fixed-axis derivative, where appropriate
($\sim$)	effective

MEASUREMENT OF DIRECT MOMENT DERIVATIVES IN THE PRESENCE OF STING PLUNGING

1.0 INTRODUCTION

It has been demonstrated that appreciable errors may be present in pitch damping derivatives determined using sting-supported models if the effects of sting plunging are not accounted for^(1,2). This is especially true in tests involving bulbous-based bodies⁽³⁾ where very thin stings have to be used to minimize the aerodynamic sting interference, and in high- α tests of aircraft configurations where significant sting deflections are unavoidable. Fortunately, unlike aerodynamic sting interference, sting plunging effects may readily be accounted for. The sting plunging phenomenon has been approached alternatively as a one degree-of-freedom (DOF) linear⁽²⁾, a 1-DOF nonlinear⁽¹⁾, or a 2-DOF linearized problem^(1,4). However, in spite of these efforts it seems that sting plunging corrections are as yet not universally applied, perhaps as they might have the appearance of being awkward to implement. For this reason a critical review of the available techniques is of interest.

In this paper it is shown that the analysis due to Burt and Uselton⁽²⁾ may be extended to obtain equations for the effect of translational motion on measured pitch damping which are in agreement with Ericsson's linear analysis⁽¹⁾ and that both of these methods can be verified.

An expression is derived for the location of the effective axis of rotation for a model undergoing a pitch-plunge oscillation and subsequently used to simplify the equations for the correction of the moment derivatives. The implementation of the required corrections in pitch/yaw oscillation experiments is discussed with particular reference to testing of aircraft configurations. Methods are suggested for determining independent corrections for the derivatives due to pitching and plunging, C_{mq} and $C_{m\dot{\alpha}}$. Although the discussion is confined to the pitch plane derivatives, the analyses are equally valid in the case of the β derivatives, $C_{nr} - C_{n\beta} \cos \alpha$ and $C_{n\dot{\beta}}$; however, the latter is usually less of a problem since the corresponding sting deflections are normally much smaller than in the pitch plane.

2.0 GENERAL CONSIDERATIONS

The effects of sting translational motion on measured stability derivatives are manifested in three different ways:

- (a) The reactions from which the moment derivatives are obtained contain translational effects.
- (b) The location of the axis of oscillation is a function of the plunge amplitude.
- (c) The model angular orientation changes with sting deflection.

Unless these effects are accounted for in the data reduction procedure, the quantities determined cannot be regarded as fixed-axis derivatives. The objective is, therefore, to determine the true single-DOF oscillatory data in the presence of sting motion.

Figure 1 illustrates the geometry of the model/sting system deflected under aerodynamic and inertial loads. Using the terminology defined in the nomenclature, the model angle of attack is

$$\alpha = \bar{\alpha} + \theta + \tan^{-1} \left[\dot{z} / (V_{\infty} \cos \bar{\alpha}) \right] \quad (1)$$

The question is now, how complete should the equations used in the correction process be? Using the Lagrangian formulation, Hanff and Orlik-Rückemann⁽⁵⁾ derived the equations of motion for a balance having two rotational DOFs and later Haberman⁽⁶⁾ developed a model of a 3-DOF balance

and included two DOFs of the supporting sting. Billingsley⁽⁷⁾ describes a treatment based on the lumped parameter concept for a multi-DOF system. Since the assumption of n DOFs results in n simultaneous second-order differential equations, which may have to be solved numerically, the additional complexity introduced by the requirement of more than two DOFs cannot be justified in routine corrections to conventionally measured direct derivatives; rather, a closed form expression of the equations of motion is a prerequisite here.

Closed form solutions may be obtained for a 2-DOF system when appropriate simplifying assumptions are made. For instance, assuming linear aerodynamics and small amplitudes, the differential equations of motion for a model oscillating in the pitch plane, where $q = \dot{\theta}$, may be written

$$\begin{aligned} I_y \ddot{\theta} + D\dot{\theta} + K\theta &= M_{\ddot{\theta}}\ddot{\theta} + M_{\dot{\theta}}\dot{\theta} + M_{\theta}\theta + M_{\ddot{z}}\ddot{z} + M_{\dot{z}}\dot{z} + M_z(t) \\ m\ddot{z} + D_z\dot{z} + K_z z &= Z_{\ddot{\theta}}\ddot{\theta} + Z_{\dot{\theta}}\dot{\theta} + Z_{\theta}\theta + Z_{\ddot{z}}\ddot{z} + Z_{\dot{z}}\dot{z} \end{aligned} \quad (2)$$

The complete 2-DOF equations may again be too complex for the present purpose. Upon further simplification, a set of coupled 2-DOF equations may be obtained, having a solution of the form

$$\begin{aligned} \theta &= \Delta\theta_0 e^{i(\omega t - \Phi)} \\ z &= \Delta z_0 e^{i\Phi_z} \left[\theta / \Delta\theta_0 \right] \end{aligned} \quad (3)$$

In reality, this motion has only one independent DOF. An equivalent but simplified form is obtained by integrating the moment equation alone and assuming a coupled- z constraint in this 1-DOF system.

$$\begin{aligned} \theta &= \Delta\theta_0 \cos(\omega t) \\ z &= \Delta z_0 \cos(\omega t + \Phi_z) \end{aligned} \quad (4)$$

Equation (4) represents the ultimate simplification of the dynamics of the system, in which the fact that the sting may move independently of the model is ignored. Nevertheless, Equation (4) could give acceptable results when the oscillation frequency is not critical ($\omega_z > 2\omega$).

3.0 REVIEW OF EXISTING TECHNIQUES

A literature survey revealed the existence of only three publications, namely those due to Ericsson⁽¹⁾, Burt and Uselton⁽²⁾ and Canu⁽⁴⁾, dealing explicitly with the analysis of sting motion in dynamic stability tests, which will be discussed in detail. The 2-DOF nature of experiments involving sting-mounted models has been recognized for some time. For instance, Thompson et al⁽⁸⁾ obtained oscillatory derivatives directly after treating the model motion as having two DOFs at the onset. However, since the equations of motion are not solved to account for the sting motion *per se*, this report is not of direct interest here. Moreover, the corrections for sting oscillations in experiments involving nonplanar motions⁽⁹⁾ are not considered here.

3.1 2-DOF Linear Analysis — Canu⁽⁴⁾

In his analysis, Canu utilizes the linear 2-DOF equations, equivalent to Equation (2), in operational notation. The equations are transformed to an axis in a frame of reference for which the 2-DOF system reduces to a single-DOF system. In so doing, the method accounts for the effect of plunging on the oscillation axis in addition to its effect on the aerodynamic reactions. The point along the longitudinal axis for which the system reduces to a single-DOF oscillation is obtained by testing at two different frequencies. Subsequently, the derivatives $C_{m\alpha}$, $C_{Z\alpha}$, $C_{mq} + C_{m\dot{\alpha}}$ and $C_{Zq} + C_{Z\dot{\alpha}}$ are transformed to the reference centre of rotation, implicitly assuming that the dynamic derivatives are independent of frequency.

Although Canu appears to have correctly accounted for the effects of sting plunging on the measured derivatives, the viability of his method has not been demonstrated, at least as far as the paper reviewed is concerned. While the data reduction equations used appear to eliminate the effect of sting plunging on the measurement of the static derivative $C_{m\alpha}$ for the AGARD-B model tested, their utility in the case of the pitch damping $C_{mq} + C_{m\dot{\alpha}}$ is not so obvious. As might be expected, the scatter in the damping data determined at different frequencies vanishes when the data are referred to the effective axis of oscillation, but it is surprising that there is little or no difference between the uncorrected (conventionally determined, 1-DOF) and corrected data.

While it is possible that the moment ascribed to the shift in the axis of rotation could balance the moment due to sting plunging at a particular frequency, this condition is not likely to prevail over the range of frequencies tested, particularly since the derivatives were assumed to be frequency independent. Moreover, although the method is suited to on-line data reduction, this advantage is negated by the requirement for testing at two frequencies. For these reasons the analysis due to Canu will not be pursued in detail.

3.2 2-DOF Linear Analysis -- Ericsson⁽¹⁾

In contrast, Ericsson's linear analysis appeared quite sound and could subsequently be verified in detail. Assuming linear aerodynamics, Ericsson obtains a general solution to the 2-DOF equations of motion (Eq. (2)) in the form of Equation (3). It is also assumed that the sting deflection angle is small, i.e. $\theta = \theta_f$ (see Fig. 1). The specific form derived by Ericsson is relevant to the present discussion, viz.

$$\begin{aligned}\theta(\tau) &= \Delta\theta e^{i(\bar{\omega}\tau - \Phi)} \\ \xi(\tau) &= \frac{\Delta\xi}{\Delta\theta} e^{i\Phi} z\theta(\tau) \\ \Delta\theta &= \frac{\Delta C_{me}}{\mu\kappa^2} \left\{ \left[\bar{\omega}_{\theta 0}^2 - \bar{\omega}^2 - \frac{C_{m\alpha}}{\mu\kappa^2} \left(1 - \frac{\bar{\omega}^2 \left(\delta_{z0} + \frac{C_{L\alpha}}{\mu} \right) \frac{C_{L\alpha}}{\mu}}{(\bar{\omega}_{z0}^2 - \bar{\omega}^2)^2 + \bar{\omega}^2 \left(\delta_{z0} + \frac{C_{L\alpha}}{\mu} \right)^2} \right) \right]^2 \right. \\ &\quad \left. + \bar{\omega}^2 \left[\delta_{\theta 0} - \frac{C_{mq}}{\mu\kappa^2} + \frac{C_{L\alpha}}{\mu} \frac{\frac{C_{m\alpha}}{\mu\kappa^2} (\bar{\omega}_{z0}^2 - \bar{\omega}^2)}{(\bar{\omega}_{z0}^2 - \bar{\omega}^2)^2 + \bar{\omega}^2 \left(\delta_{z0} + \frac{C_{L\alpha}}{\mu} \right)^2} \right]^2 \right\}^{-1/2}\end{aligned}\quad (5)$$

$$\Phi = \sin^{-1} \left\{ \frac{\Delta\theta \mu \kappa^2 \bar{\omega}}{\Delta C_{me}} \left[\delta_{\theta 0} - \frac{C_{mq}}{\mu \kappa^2} + \frac{\frac{C_{L\alpha} C_{m\alpha}}{\mu^2 \kappa^2} (\bar{\omega}_{z0}^2 - \bar{\omega}^2)}{(\bar{\omega}_{z0}^2 - \bar{\omega}^2)^2 + \bar{\omega}^2 \left(\delta_{z0} + \frac{C_{L\alpha}}{\mu} \right)^2} \right] \right\}$$

$$\frac{\Delta \xi}{\Delta \theta} = \frac{C_{L\alpha}}{\mu} \left[\frac{1 + \bar{\omega}^2 \left(\frac{C_{Lq}}{C_{L\alpha}} \right)^2}{(\bar{\omega}_{z0}^2 - \bar{\omega}^2)^2 + \bar{\omega}^2 \left(\delta_{z0} + \frac{C_{L\alpha}}{\mu} \right)^2} \right]^{1/2} \quad (5)$$

(cont'd)

$$\Phi_z = \sin^{-1} \left\{ \frac{\bar{\omega} \left[\left(\delta_{z0} + \frac{C_{L\alpha}}{\mu} \right) - \frac{C_{Lq}}{C_{L\alpha}} (\bar{\omega}_{z0}^2 - \bar{\omega}^2) \right]}{\left\{ \left[1 + \bar{\omega}^2 \left(\frac{C_{Lq}}{C_{L\alpha}} \right)^2 \right] \left[(\bar{\omega}_{z0}^2 - \bar{\omega}^2)^2 + \bar{\omega}^2 \left(\delta_{z0} + \frac{C_{L\alpha}}{\mu} \right)^2 \right] \right\}^{1/2}} \right\}$$

The analysis is then specialized to the case of low lift configurations, assuming that the aerodynamic as well as mechanical damping are negligible compared with the critical damping, to obtain the following expressions† for the moment derivatives in terms of wind-on and wind-off conditions

$$C_{mqz} = C_{mq} - \frac{\Delta \xi}{\Delta \theta} C_{m\alpha} (\bar{\omega}_{z0}^2 - \bar{\omega}^2) \left[1 + \bar{\omega}^2 \left(\frac{C_{Lq}}{C_{L\alpha}} \right)^2 \right]^{-1/2}$$

$$\left[(\bar{\omega}_{z0}^2 - \bar{\omega}^2)^2 + \bar{\omega}^2 \left(\delta_{z0} + \frac{C_{L\alpha}}{\mu} \right)^2 \right]^{-1/2} \quad (6)$$

$$= \left(\frac{\Delta C_{m0}}{\Delta \theta \bar{\omega}} \sin \Phi \right)_{\text{OFF}} \Omega - \left(\frac{\Delta C_{m0}}{\Delta \theta \bar{\omega}} \sin \Phi \right)_{\text{ON}}$$

† If the factor 2 is included in the nondimensional forms, e.g. $q\bar{c}/(2V_\infty)$, which is more common practice, corresponding factors of 2 will appear in the equations for C_{mq} .

$$C_{m\theta z} = C_{m\alpha} \left[1 - \left(\frac{\Delta \xi}{\Delta \theta} \right)^2 \bar{\omega}^2 \frac{1 + \frac{\mu \delta_{zo}}{C_{L\alpha}}}{1 + \bar{\omega}^2 \left(\frac{C_{Lq}}{C_{L\alpha}} \right)^2} \right] \quad (6)$$

$$= \mu \kappa^2 \left[(\bar{\omega}^2)_{OFF} - (\bar{\omega}^2)_{ON} \right] - \left(\frac{\Delta C_{mo}}{\Delta \theta} \cos \Phi \right)_{ON} + \left(\frac{\Delta C_{mo}}{\Delta \theta} \cos \Phi \right)_{OFF} \quad (\text{cont'd})$$

In further simplifying Equation (6), a somewhat ambiguous part of Ericsson's paper followed. Accordingly, a comment is included here in the interests of clarification. Returning to Equation (5), it can be shown that

$$\cos \Phi_z = \frac{(\bar{\omega}_{zo}^2 - \bar{\omega}^2) + \bar{\omega}^2 \frac{C_{Lq}}{C_{L\alpha}} \left(\delta_{zo} + \frac{C_{L\alpha}}{\mu} \right)}{\left[1 + \bar{\omega}^2 \left(\frac{C_{Lq}}{C_{L\alpha}} \right)^2 \right]^{1/2} \left[(\bar{\omega}_{zo}^2 - \bar{\omega}^2)^2 + \bar{\omega}^2 \left(\delta_{zo} + \frac{C_{L\alpha}}{\mu} \right)^2 \right]^{1/2}} \quad (7)$$

Since the structural and aerodynamic damping have been assumed to be small, $\delta_{zo} + C_{L\alpha}/\mu = 0[\bar{\omega}^2]$ and $C_{Lq}/C_{L\alpha} < 1$. Then for $\bar{\omega}_{zo} \neq \bar{\omega}$,

$$\cos \Phi_z = \frac{\bar{\omega}_{zo}^2 - \bar{\omega}^2}{\left[(\bar{\omega}_{zo}^2 - \bar{\omega}^2)^2 \right]^{1/2}} \quad (8)$$

where terms of $O[\bar{\omega}^4]$ have been neglected. The Equations (6) yields

$$C_{mqz} = C_{mq} - \frac{\Delta \xi}{\Delta \theta} C_{m\alpha} \cos \Phi_z \quad (9)$$

Equations (8) and (9) will be in agreement with the results presented by Ericsson (Ref. 1, Eq. [11]) if the signs of $\cos \Phi_z$ and of the term $(\Delta \xi / \Delta \theta) C_{m\alpha} \cos \Phi_z$ are changed. The net result is, of course, the same in the two cases. The correctness of Equations (8) and (9) may be verified by returning to Equations (5), whence it may be shown that

$$\frac{\Delta \xi}{\Delta \theta} \cos \Phi_z = \frac{C_{L\alpha}}{\mu} \frac{(\bar{\omega}_{zo}^2 - \bar{\omega}^2) \left[1 + \frac{\bar{\omega}^2}{\bar{\omega}_{zo}^2 - \bar{\omega}^2} \frac{C_{Lq}}{C_{L\alpha}} \left(\delta_{zo} + \frac{C_{L\alpha}}{\mu} \right) \right]}{(\bar{\omega}_{zo}^2 - \bar{\omega}^2)^2 + \bar{\omega}^2 \left(\delta_{zo} + \frac{C_{L\alpha}}{\mu} \right)^2} \quad (10)$$

When substituted in Ericsson's Equation [9] and using the above assumptions, Equation (9) is again obtained. Also note that the values of Φ_z obtained from Equation (8), $\Phi_z = 0$ for $\bar{\omega}_{z0} > \bar{\omega}$ and $\Phi_z = \pi$ for $\bar{\omega}_{z0} < \bar{\omega}$, differ from those given by Ericsson.

The correct form of the approximate equations for the effect of plunging on the fixed-axis moment derivatives is then

$$C_{mqz} = C_{mq} - \frac{\Delta\xi}{\Delta\theta} C_{m\alpha} \cos \Phi_z$$

$$C_{m\theta z} \approx C_{m\alpha} \left[1 - \bar{\omega}^2 \left(\frac{\Delta\xi}{\Delta\theta} \right)^2 \right]$$
(11)

$$\frac{\Delta\xi}{\Delta\theta} = -\frac{C_{L\alpha}}{\mu} \bigg/ |\bar{\omega}_{z0}^2 - \bar{\omega}^2|$$

$$\Phi_z \approx \cos^{-1} \left\{ \frac{\bar{\omega}_{z0}^2 - \bar{\omega}^2}{|\bar{\omega}_{z0}^2 - \bar{\omega}^2|} \right\} = \begin{cases} 0 & ; \bar{\omega}_{z0} > \bar{\omega} \\ \pi & ; \bar{\omega}_{z0} < \bar{\omega} \end{cases}$$

Thus, with the exception of the small discrepancy noted, the entire linear analysis due to Ericsson has been corroborated. In summary, Equation (6) may be described as a solution to a coupled pitch-plunge system applicable to low-lift configurations when the amplitudes and reduced frequencies are small and the damping negligible. On the other hand, the approximate Equations (11) represent the solution to a single-DOF system of a model oscillating in pitch subject to an equation of constraint for the plunge mode, of the form of Equation (4), which is implicitly restricted to noncritical sting natural frequencies. Implementation of Equation (11) could be based on measurements of ω and ω_{z0} , or preferably, $\Delta\xi$, $\Delta\theta$ and Φ_z , using appropriate sting instrumentation.

Ericsson's conclusions are also correct; for instance, that for subcritical sting stiffness, the sting plunging will increase the measured damping C_{mq} and decrease the static stability. This observation agrees qualitatively with the results presented by Burt and Uselton⁽²⁾ if the sting stiffness used in that test was, in fact, subcritical. However, it should be noted that neither Ericsson nor Burt and Uselton accounted for the shift in the axis of oscillation and therefore the comparison is inconclusive.

3.3 1-DOF Linear Analysis — Burt and Uselton⁽²⁾

The basic premise in this analysis is that the lift forces may be ignored so that only the moment equation need be considered. The plunging enters this equation only in its effects on the angle of attack (Eq. (1)) and in a mass unbalance term. The single-DOF solution therefore consists of a linear pitch equation and a plunge equation of constraint similar to Equation (4). Then, setting in addition, $M_e = \Delta M_e \cos(\omega t + \Phi)$, and assuming the sting frequency is noncritical ($\sin \Phi_z = 0$), Burt and Uselton solve for the derivatives to obtain

$$M_q + M_{\dot{\alpha}} = \frac{\frac{M_{\alpha}}{V_{\infty} \cos \bar{\alpha}} \frac{\Delta z \cos \Phi_z}{\Delta \theta_f} + D - \frac{\Delta M_c \sin \Phi}{\omega \Delta \theta_f}}{1 + \frac{\Delta \theta_z}{\Delta \theta_f} \cos \Phi_z} \quad (12)$$

$$-M_{\alpha} = K \left[\left(\frac{\omega}{\omega_{\theta_0}} \right)^2 - 1 \right] + \frac{K \frac{\Delta \theta_z}{\Delta \theta_f} \cos \Phi_z - \delta K + \frac{\Delta M_c}{\Delta \theta_f} \cos \Phi + m x_{cg} \omega^2 \frac{\Delta z}{\Delta \theta_f} \cos \Phi_z}{1 + \frac{\Delta \theta_z}{\Delta \theta_f} \cos \Phi_z}$$

A term in $M_{\dot{\alpha}}$ has been neglected in the expression for M_{α} .

The sting oscillation parameters $\Delta \theta_z \cos \Phi_z / \Delta \theta_f$ and $\Delta z \cos \Phi_z / \Delta \theta_f$ may be expressed in terms of the dynamic moments sensed by the sting and balance instrumentation and the sting deflection constants.

$$\frac{\Delta z}{\Delta \theta_f} \cos \Phi_z = -K \left(\frac{\partial z}{\partial M_p} \right)_{M_s} - \left(\frac{\Delta M_s}{\Delta \theta_f} \cos \Phi_z \right) \left(\frac{\partial z}{\partial M_s} \right)_{M_p}$$

$$\frac{\Delta \theta_z}{\Delta \theta_f} \cos \Phi_z = K \left(\frac{\partial \theta_z}{\partial M_p} \right)_{M_s} + \left(\frac{\Delta M_s}{\Delta \theta_f} \cos \Phi_z \right) \left(\frac{\partial \theta_z}{\partial M_s} \right)_{M_p} \quad (13)$$

For the case where the sting is not instrumented, expressions were derived for the sting oscillation parameters in terms of the dynamic force and moment at the pivot, assuming that static force data are available. Thus, in this case, the force equation of motion is introduced after the solution has been obtained. These expressions were verified in detail together with the foregoing analysis but, being quite cumbersome, the equations will not be repeated here.

Since the mechanical damping, stiffness and mass unbalance terms appear explicitly in Equation (12) the sting plunging corrections may be applied without additional tare measurements provided appropriate calibration tests are performed. The quantities to be determined include the balance and sting calibration constants, structural stiffness and damping and the sting deflection constants. Therefore, although this is essentially a simple method, the requirement for preparatory measurements could be considerable. As shown in the next section, Equation (12) can be reduced to a form compatible with Ericsson's results, which is more conveniently incorporated in an existing data reduction procedure.

Strictly speaking, the method is not applicable to aircraft configurations since it is based on a single-DOF solution which ignores lift forces. Nevertheless, for sufficiently high sting frequencies ($\omega_{z_0} > 3\omega$) the approximation could be acceptable. This should be borne in mind when considering the AGARD-C data presented by Burt and Uselton. Moreover, as pointed out before, the analysis under discussion does not account for the movement of the effective axis of oscillation. Therefore, the possibility that the good agreement reported between the corrected $C_{m\alpha}$ and their static test counterparts might be fortuitous, cannot be ruled out. More specifically, it is shown in the next section that Burt and Uselton's conclusion that sting plunging will lead to deficient static stability measurements is circumstantial, and probably originated in the effects of a mass unbalance.

3.4 1-DOF Nonlinear Analysis -- Ericsson⁽¹⁾

When the oscillation amplitude is not small and significant aerodynamic nonlinearities are present, the analyses described above could be quite inadequate. Ericsson addresses this problem for the case of a blunt cylinder-flare body, assuming that the lift is locally linear and the pitching moment can be approximated by

$$C_m = \frac{\alpha}{|\alpha|} \Delta C_{ms} + C_{m\alpha} \alpha \quad (14)$$

A single-DOF motion of the Form (4) is assumed and, upon evaluating the energy integral for one oscillation cycle, the following result is obtained for the effective damping coefficient

$$\tilde{C}_{mqz} = \tilde{C}_{mq} - \frac{\Delta \xi}{\Delta \theta} C_{m\alpha} \cos \Phi_z \left[1 + \frac{4}{\pi} \frac{\alpha^*}{\Delta \theta} \left(1 - \left(\frac{\bar{\alpha}}{\Delta \theta} \right)^2 \right)^{1/2} \right] \quad (15)$$

where $\alpha^* = \Delta C_{ms}/C_{m\alpha}$ and $\cos \Phi_z$ is defined in Equation (11) (note that the same discrepancy is present here regarding the sign of $\cos \Phi_z$). For obvious reasons C_{mqz} cannot be obtained in this manner.

This result is based on the same set of simplifying assumptions common to the linear methods; only the restriction of linear aerodynamics has been relaxed to accommodate an assumed form of the nonlinearity in C_m . Therefore, when the nonlinearity is removed ($\Delta C_{ms} = 0$), Equation (15) should reduce to Equation (11), which it in fact does. However, unlike Equation (11), Equation (15) ignores the lift forces on the model, a circumstance which has its origin in the constraint of 1-DOF as in Equation (4). This solution is, therefore, only valid for a rather special case.

Two conclusions may be drawn from these observations. Firstly, the nonlinear approach described may not have any real advantage over the approximate linear corrections (Eq. (11)) in tests at high angles of attack, where the lift coefficient is likely to be highly nonlinear. Secondly, it is expected that in most instances involving aircraft configurations, the 2-DOF linear equations (Eq. (6)) could be more useful than the 1-DOF nonlinear analysis in its present form. These conclusions notwithstanding, Ericsson's results provide a valuable demonstration that the effect of sting plunging can be larger in the nonlinear case.

3.5 Summary of Restrictions

In summary, the linearized analyses due to Ericsson⁽¹⁾ and Burt and Uselton⁽²⁾ are particularly useful within their legitimate bounds of applicability and are suitable for on-line data reduction. Certain assumptions are common to all of the methods considered, including rigid mounting, structural damping small and independent of static loads and frequency, and low lift and oscillation frequencies. A summary of restrictions is presented in Table 1. Only the 2-DOF analyses of Ericsson⁽¹⁾ and Canu⁽⁴⁾ are applicable at near-critical sting frequencies and, with the exception of Ericsson's nonlinear analysis, the methods are based on linear aerodynamics and assumed small oscillation amplitudes. Canu's analysis determines the effective axis of oscillation but this is not discussed in the other two papers. The effective axis of rotation may be determined geometrically⁽¹⁰⁾; an appropriate method is introduced below.

4.0 EXTENSION OF THE ANALYSIS

4.1 Comparison of the 1-DOF and 2-DOF Linear Methods

The utility of the analysis due to Burt and Uselton⁽²⁾ can be enhanced through manipulation of Equation (12). First of all, assuming that $\Delta\theta_z \ll \Delta\theta_f$, (i.e. $\Delta\theta_f = \Delta\theta$), the following relations may be written for wind-on and wind-off conditions:

$$M_q + M_{\alpha} - \frac{M_{\alpha}}{V_{\infty} \cos \bar{\alpha}} \frac{\Delta z \cos \Phi_z}{\Delta \theta} - D = - \left(\frac{\Delta M_c}{\omega \Delta \theta} \sin \Phi \right)_{ON} \quad (16)$$

$$D = \left(\frac{\Delta M_c}{\omega \Delta \theta} \sin \Phi \right)_{OFF} \frac{\omega_{OFF}}{\omega_{ON}}$$

since D was assumed to be unaffected by static loads. Then, writing $\bar{M}_q$ for $M_q + M_{\alpha}$,

$$\bar{M}_{qz} = \left(\frac{\Delta M_c}{\omega \Delta \theta} \sin \Phi \right)_{OFF} \frac{\omega_{OFF}}{\omega_{ON}} - \left(\frac{\Delta M_c}{\omega \Delta \theta} \sin \Phi \right)_{ON}$$

i.e.

$$\bar{M}_{qz} = \bar{M}_q - \frac{M_{\alpha}}{V_{\infty} \cos \bar{\alpha}} \frac{\Delta z}{\Delta \theta} \cos \Phi_z \quad (17)$$

Similarly, if the variation of stiffness with applied force may be ignored, $\delta K = 0$ and

$$M_{\alpha} + \frac{K}{\omega_{\theta 0}^2} \left[\omega_{ON}^2 - \omega_{\theta 0}^2 \right] + m x_{cg} \omega_{ON}^2 \frac{\Delta z}{\Delta \theta} \cos \Phi_z = - \left(\frac{\Delta M_c}{\Delta \theta} \cos \Phi \right)_{ON}$$

Then, since $\omega_{\theta 0}^2 = K/I_y$

$$M_{\alpha} + \omega_{ON}^2 \left[I_y + m x_{cg} \frac{\Delta z}{\Delta \theta} \cos \Phi_z \right] - K = - \left(\frac{\Delta M_c}{\Delta \theta} \cos \Phi \right)_{ON} \quad (18)$$

$$\omega_{OFF}^2 \left[I_y + m x_{cg} \frac{\Delta z}{\Delta \theta} \cos \Phi_z \right] - K = - \left(\frac{\Delta M_c}{\Delta \theta} \cos \Phi \right)_{OFF}$$

Then

$$M_{\alpha} + m x_{cg} \frac{\Delta z}{\Delta \theta} \cos \Phi_z (\omega_{ON}^2 - \omega_{OFF}^2) = I_y (\omega_{OFF}^2 - \omega_{ON}^2) + \left(\frac{\Delta M_e}{\Delta \theta} \cos \Phi \right)_{OFF} - \left(\frac{\Delta M_e}{\Delta \theta} \cos \Phi \right)_{ON}$$

i.e.

$$M_{\theta z} = M_{\alpha} + m x_{cg} \frac{\Delta z}{\Delta \theta} \cos \Phi_z (\omega_{ON}^2 - \omega_{OFF}^2) \quad (19)$$

Hence, the only contribution to $M_{\theta z} - M_{\alpha}$ is that due to unbalance. The fact that Equation (19) is independent of aerodynamic forces is the consequence of neglecting such forces in the equation of motion. Moreover, the important conclusion drawn by Ericsson⁽¹⁾ that sting plunging will, in general, lead to deficient static stability measurements cannot be deduced from the analysis of Burt and Uselton.

On the other hand, in the case of the damping derivative it may be shown that there is a measure of agreement between the two techniques. In coefficient form, Equation (17) becomes

$$\bar{C}_{mqz} = \bar{C}_{mq} - \frac{\Delta \xi}{\Delta \theta} C_{m\alpha} \cos \Phi_z \quad (20)$$

which is in agreement with, although less complete than, Ericsson's approximate equation (Eq. (11)). Unlike its counterpart in Equation (11), $\Delta \xi / \Delta \theta$ in Equation (20) does not explicitly contain $C_{L\alpha}$. The following result is obtained from Equation (19).

$$\begin{aligned} C_{m\theta z} &= C_{m\alpha} - 2\mu \xi_{cg} (\bar{\omega}_{OFF}^2 - \bar{\omega}_{ON}^2) \frac{\Delta \xi}{\Delta \theta} \cos \Phi_z \\ &= 2\mu \kappa^2 (\bar{\omega}_{OFF}^2 - \bar{\omega}_{ON}^2) + \left(\frac{\Delta C_{me}}{\Delta \theta} \cos \Phi \right)_{OFF} - \left(\frac{\Delta C_{me}}{\Delta \theta} \cos \Phi \right)_{ON} \end{aligned} \quad (21)$$

Equation (21) reduces to the form of Equation (6) for the trivial case where $\xi_{cg} = 0$ and $C_{Lq} = C_{L\alpha} = 0$. Hence it may be concluded that the methods due to Ericsson⁽¹⁾ and Burt and Uselton⁽²⁾ will yield the same results at identical levels of approximation.

4.2 Transformation to the Effective Oscillation Axis

The effective axis of oscillation in a pitch-plunge motion may be derived from the sting-model geometry depicted in Figure 2. Using the pivot axis as the origin, the location of the axis of rotation, x_c , is obtained in terms of the angular deflection and the displacement Δz_a of a point x_a on the model axis (which could be the location of an accelerometer).

$$\frac{\Delta z}{x_c} = \frac{\Delta z_a}{x_c - x_a} = \sin(\Delta\theta_f + \Delta\theta_z) \quad (22)$$

whence

$$x_c = \Delta z_a / \sin(\Delta\theta_f + \Delta\theta_z) + x_a \quad (23)$$

Therefore, the rotation axis is displaced a distance x_c along the model axis given by Equation (23).

Note that x_c could be measured directly, for instance, by optical means. When such measurements are sufficiently accurate, the following simplification becomes possible. Assuming that $\theta_z \simeq 0$, which is perfectly reasonable when the sting is relatively rigid as in most dynamic stability testing, and since $\Delta\theta \simeq \Delta\theta_f$ is small, Equation (22) yields

$$\frac{\Delta\zeta}{\Delta\theta} = \frac{x_c}{\bar{c}} = \xi_c \quad (24)$$

Then Equation (11) reduces to the simple form

$$C_{mq\dot{z}} = C_{mq} \pm \xi_c C_{m\alpha} \begin{cases} \bar{\omega}_{zo} < \bar{\omega} \\ \bar{\omega}_{zo} > \bar{\omega} \end{cases} \quad (25)$$

$$C_{m\theta z} = C_{m\alpha} [1 - (\xi_c \bar{\omega})^2]$$

Equation (25) does not involve any new assumptions since $\Delta\theta = \Delta\theta_f$ is already implicit in Equation (11). It is, therefore, possible to apply approximate corrections for the effects of sting plunging, even when no sting instrumentation is available; however, the practicability of direct measurements of x_c might be limited by experimental constraints. Consider, for instance, the expedient of visually locating a nontranslating point on the model. The reading could be taken from a scale on the side of the model to an accuracy of perhaps ± 0.5 mm, provided that x_c does not fall beyond the extremities of the fuselage or wing tip. The restrictions associated with this approach are included in Table 1.

Normally, it will be possible to balance the model well enough that any residual mass offset might be ignored. When a large unbalance has to be contented with, the effects on $C_{m\alpha}$ might be estimated from the following expression derived from Equation (21)

$$C_{m\theta z} = C_{m\alpha} \pm 2\mu\xi_{cg}\xi_c (\bar{\omega}_{OFF}^2 - \bar{\omega}_{ON}^2) \begin{cases} \bar{\omega}_{zo} < \bar{\omega} \\ \bar{\omega}_{zo} > \bar{\omega} \end{cases} \quad (26)$$

Subsequently, the measured moment derivatives may be transformed to the reference centre at the pivot using the standard relations for transformations between different axes⁽¹¹⁾. However, this procedure is unsatisfactory in the case of the dynamic derivatives, introducing a requirement for additional tests at different centres of rotation.

4.3 Dynamic Testing of Aircraft Configurations

Translational acceleration and dynamic coupling forces and moments become significant in dynamic tests of aircraft models at high angles of attack, necessitating more complete analyses of the sting-model motion. It would appear that, in most cases, a 2-DOF solution valid for high-lift conditions would be adequate in the data reduction for direct derivatives. This implies that, for instance, $C_{m\dot{\alpha}}$ would be formulated independently of C_{mq} and, therefore, also individually corrected for sting motion. In the presence of pronounced nonlinearities, such a 2-DOF solution could be inadequate and if nonlinear aerodynamics should be incorporated, it is likely that a numerical solution would be required. Thus, an on-line correction for sting oscillation might be ruled out in general high- α aircraft tests, even for the case of the direct moment derivatives.

The reliability of closed-form solutions for the estimation of sting motion effects is further impaired by other effects which cannot easily be modelled. For instance, all of these methods yield steady state solutions where $\bar{\omega}$, $\Delta\theta$ and $\Delta\zeta$ are constant for a particular test situation. This is quite far from reality in high- α aircraft tests, where the response may be of a highly transient nature due to dynamic flow behaviour. Such a situation could, perhaps, only be handled through real-time numerical integration, to continuously correct the deflection vectors sensed by the balance.

In spite of these complications, it might still be possible to obtain reasonable estimates of sting plunging effects from simplified expressions. For example, extending the analysis of Reference 2, the following relationship may be derived

$$-M_{\dot{\alpha}} = \frac{V_{\infty} \cos \bar{\alpha}}{\omega^2} \left\{ \frac{K \left(\frac{\omega^2}{\omega_{\theta 0}^2} - 1 \right) + M_{\alpha} + \frac{\Delta M_c}{\Delta \theta} \cos \Phi}{\frac{\Delta z}{\Delta \theta} \cos \Phi_z} + x_{cg} m \omega^2 \right\} \quad (27)$$

This suggests that a correction to $C_{m\dot{\alpha}}$ could be obtained in terms of $C_{m\alpha}$. However, it is not clear whether reasonable accuracy could be achieved with a procedure based on Equation (27).

A discussion of cross-coupling and acceleration derivative measurements is beyond the scope of this paper. Nevertheless, it should be noted here that there is a singular lack of knowledge of sting interference/interaction effects on such measurements. The problem could be approached using generalized co-ordinates but since a minimum of three DOFs would be involved, closed-form solutions will again be ruled out. Instead, the sting motion should be accounted for in the data reduction equations obtained from a formulation such as that due to Haberman⁽⁶⁾.

4.4 Alleviation of the Sting Oscillation Problem

The complete elimination of motion of a cantilever beam subjected to oscillatory loads is clearly impossible; nevertheless, there is considerable scope for minimizing sting oscillations. Firstly, consider the contribution of sting plunging to the damping moment. It follows from Equation (11) that

$$\frac{C_{mqz} - C_{mq}}{C_{mq}} = - \frac{C_{m\alpha}}{C_{mq}} \frac{C_{L\alpha}}{\mu} \frac{1}{|\bar{\omega}_{z0}^2 - \bar{\omega}^2|} \frac{\bar{\omega}_{z0}^2 - \bar{\omega}^2}{|\bar{\omega}_{z0}^2 - \bar{\omega}^2|} \quad (28)$$

For a subcritical sting and stable model, the minimum damping contribution is achieved as $\bar{\omega}_{z_0} \rightarrow 0$; this becomes

$$\frac{C_{mqz} - C_{mq}}{C_{mq}} = \frac{C_{m\alpha}}{C_{mq}} \frac{C_{L\alpha}}{\mu\bar{\omega}^2}$$

On the other hand, at an arbitrary supercritical sting frequency, $\bar{\omega}_{z_0} = c\bar{\omega}$ where $c > \sqrt{2}$; Equation (28) yields

$$\frac{C_{mqz} - C_{mq}}{C_{mq}} = - \frac{C_{m\alpha}}{C_{mq}} \frac{C_{L\alpha}}{\mu\bar{\omega}^2(c^2 - 1)}$$

Therefore, the undamping contribution due to plunging of a supercritical sting is always smaller than the subcritical damping contribution, provided that $\bar{\omega}_{z_0} > \sqrt{2}\bar{\omega}$. When $\bar{\omega}_{z_0} > 3\bar{\omega}$, which is fairly common in dynamic stability tests involving acceleration or cross coupling derivative measurements, the ratio is smaller than 1/8. Thus, by designing for maximum sting stiffness compatible with permissible sting/model base diameter ratios, it may be possible to obviate the need for sting plunging corrections, at least in the case of the direct derivatives.

Finally, it is noted here that the application of certain new concepts for dynamic stability testing⁽⁹⁾ could, to all intents and purposes, eliminate sting oscillations in the pitch-yaw oscillation mode. The principle of *orbital fixed-plane motion* makes it possible, *inter alia*, to generate pure pitching and yawing motions. Under these conditions the angles of attack and sideslip are invariant, which means that the static aerodynamic forces produce a fixed static deflection relative to body axes, and only the second-order, dynamic forces and moments would tend to contribute to sting oscillation. The implementation of this concept could, therefore, provide a solution to the problem of sting oscillation effects on the cross and cross-coupling derivatives due to pitching and yawing as well as the corresponding direct derivatives.

5.0 CONCLUSIONS

The contributions to the moment derivatives measured in small-amplitude pitch/yaw oscillation tests due to sting plunging are, in general, reliably accounted for in Ericsson's linear analysis for low-lift configurations⁽¹⁾. The more simplified analysis due to Burt and Uselton⁽²⁾ could also yield acceptable results for low-lift configurations and noncritical sting frequencies ($\omega_{z_0} > 3\omega$). These two methods reduce to the same result at identical levels of approximation. The approximate form of Ericsson's equations are particularly useful in on-line data reduction; the accuracy of the results so obtained would be increased if the sting deflection parameters could be measured (suitable sting instrumentation would comprise a strain gauge bridge and one accelerometer).

It is important that all of the effects due to sting motion are accounted for; namely, the effects on the derivatives, the effective axis of oscillation and mean angle of attack for which they are determined. The location of the effective axis of rotation of a model executing pitch-plunge oscillations may be determined by the method described above, using the measured sting deflection parameters. Subsequently, the measured derivatives may be transformed to the reference centre.

It is shown that the introduction of the effective axis of oscillation into Ericsson's approximate equations yields a simple form which may be implemented even when no sting instrumentation is available. The correction of the measured derivatives, as well as their transformation to the reference centre, can then be accomplished if an accurate external measurement of the axis of rotation can be made.

Although the methods discussed are, strictly speaking, not applicable to aircraft configurations, it is expected that, judiciously implemented, Ericsson's complete equations will be acceptable in most test situations. Moreover, when significant nonlinearities are present, Ericsson's nonlinear analysis might be used with an appropriate, assumed form of the nonlinear pitching moment. However, it should be noted that most of the assumptions implicit in these techniques tend to be violated in high- α testing and it is, therefore, recommended that further research be undertaken to obtain a more general description of the phenomenon, even if its ultimate form is not conducive to on-line data reduction. The altogether greater complexity of data reduction procedures based on multi-DOF models of the sting-balance-model system must be considered necessary in future forced oscillation experiments designed to obtain more accurate measurements of dynamic cross and cross-coupling derivatives.

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TABLE 1
Summary of Restrictions

METHOD RESTRICTIONS /REQUIREMENTS	CANU (4)	BURT and USELTON (2) Eq. (12)	ERICSSON (linear) (1) Eq. (6)	PRESENT METHOD Eq. (25)	ERICSSON (nonlinear) (1) Eq. (15)
Eqs. of Motion DOFs	2	1	2	1 or 2	1
Pitching Moment	l i n e a r				nonlinear
Pitch Damping	independent of frequency	negligible fraction of critical damping			
Dynamic Lift	small	ignored	small	i g n o r e d	
Model Amplitude	s m a l l , c o n s t a n t				arbitrary, constant
Sting Pitch Amplitude	negligible	arbitrary, constant	n e g l i g i b l e		
Sting Frequency	noncritical		arbitrary	noncritical	
Mass Offset	negligible	arbitrary	n e g l i g i b l e		
Symmetry	a r b i t r a r y s h a p e			axisymmetrical	
Axis Transformation	included	n o n e		included	none
Test/Measurements	2 frequencies	sting deflection parameters		axis of rotation	sting defl. parameters
Common Restrictions	low, linear lift; rigid support, structural damping small & independent of static loads; low reduced frequencies; constant sting plunge amplitude.				

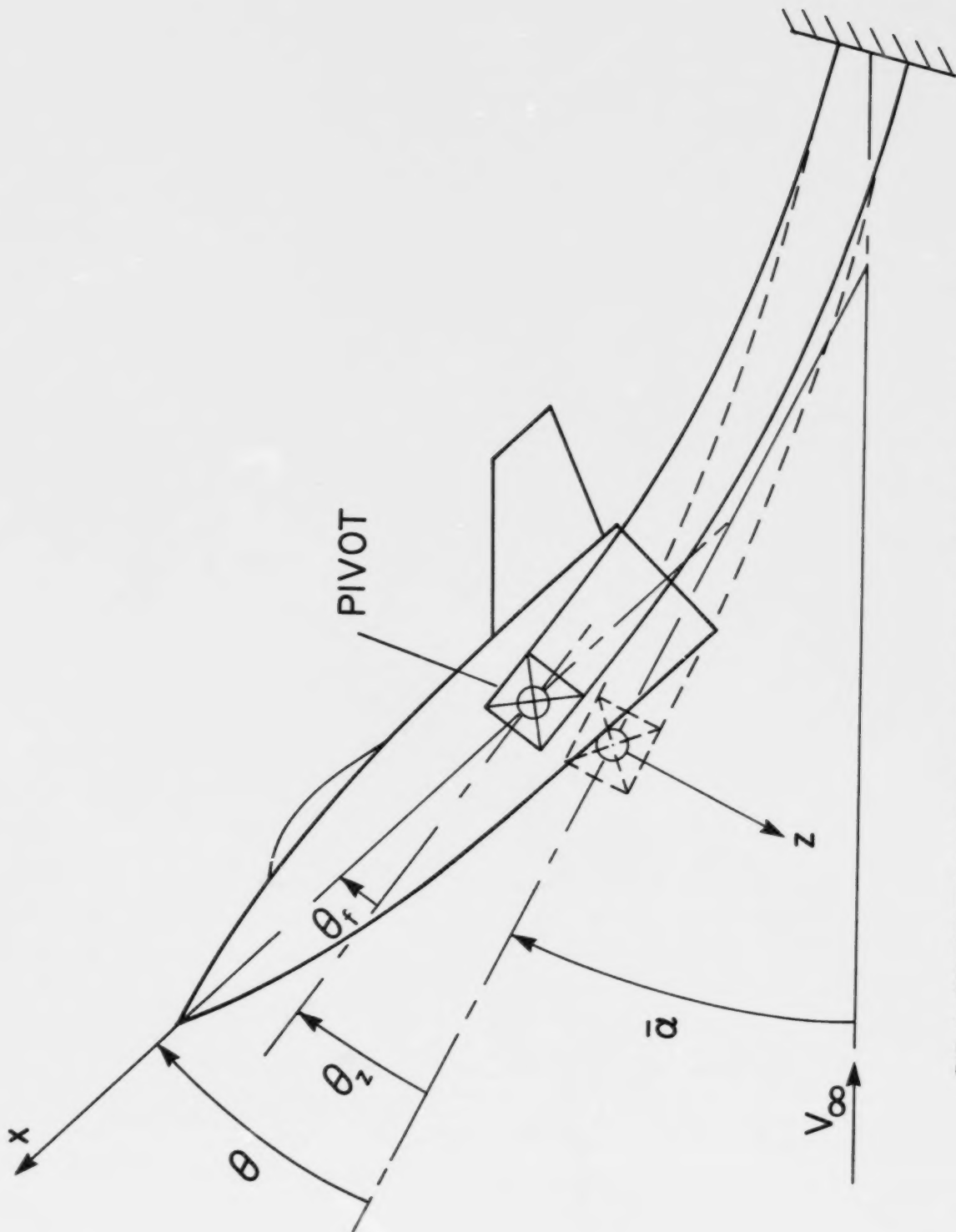


FIG. 1: SCHEMATIC ILLUSTRATING PLANAR MOTION OF STING-MODEL SYSTEM

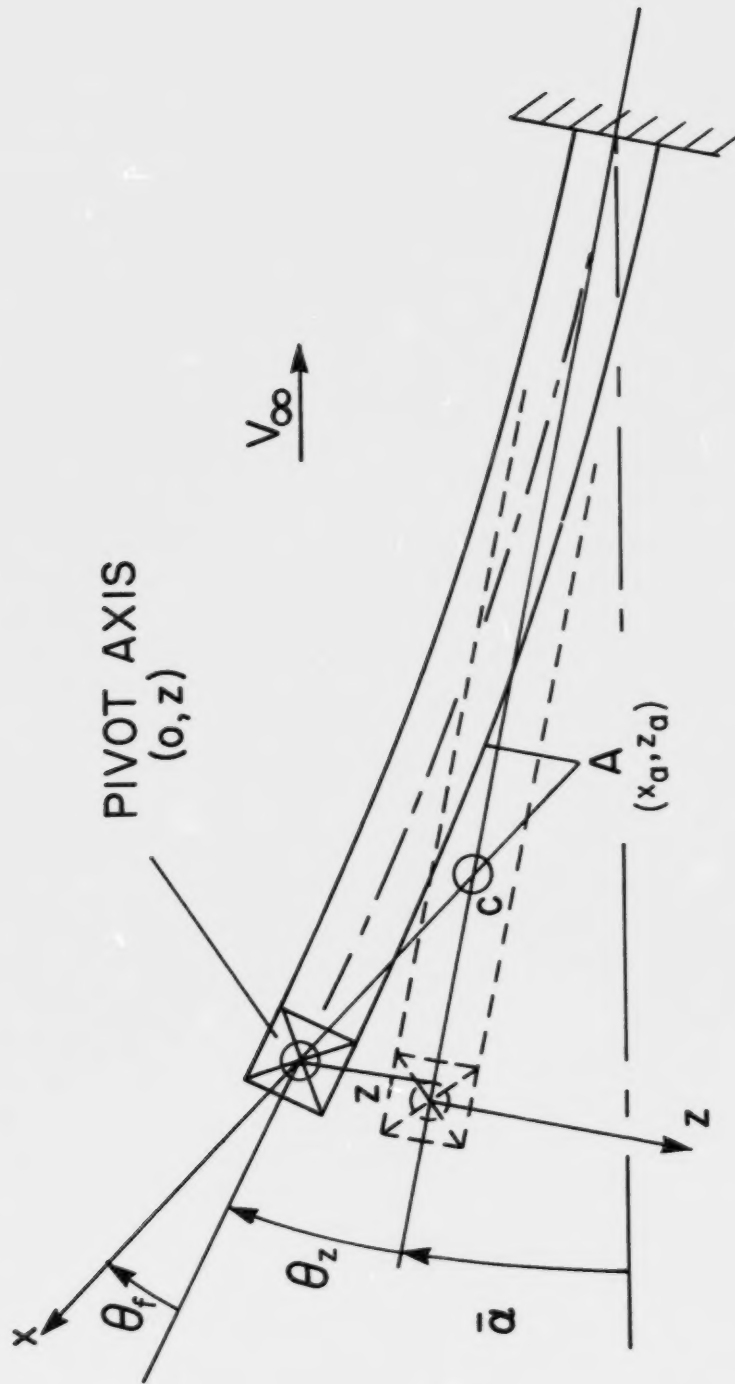


FIG. 2: EFFECTIVE AXIS OF OSCILLATION

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SUMMARY/SOMMAIRE This paper presents a review and extension of methods for determining the effects of sting oscillations on the measurement of dynamic moment derivatives, based on linear as well as nonlinear models. It is shown that two of the linearized methods will reduce to the same result, applicable to low-lift configurations and suitable for on-line data reduction. The location of the effective axis of rotation is determined for a model executing planar oscillations in two degrees of freedom. The equations for the effect of plunging on the derivatives due to angular oscillation can be simplified by relating the sting deflection parameters to the co-ordinate of this axis. Therefore, both the correction of the measured derivatives and their transformation to the reference centre can be accomplished after a single measurement is made, namely the location of the effective axis. The requirements for performing sting plunging corrections for aircraft configurations at high angles of attack are discussed. 15				